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Reliability & Variation Research

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SAMPLE SIZE FORMULAS

1. VALID LIFE TESTS

Wanted: B_Q Life accurate within ±(100 P)% with ONE-SIDED CONFIDENCE C.

Given or Estimable:
- Minimum Life α
- Weibull Slope b
- Characteristic Life θ

SAMPLE SIZE FORMULA

$$N = \left(\frac{t_C \sqrt{Q(1 - Q)}}{P \, B_Q \, f_Q} \right)^2$$

t_C = C-level t-score in a normal distribution

f_Q = Weibull Ordinate at B_Q life = $\dfrac{b(B_Q - \alpha)^{b-1}}{(\theta - \alpha)^b} e^{-\left(\frac{B_Q - \alpha}{\theta - \alpha}\right)^b}$

B_Q = B_Q Life (i.e., time in service at which fraction Q of the population will be failed.)

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2. VALID BOGEY TESTS WITH ZERO DEFECTIVES

Wanted : Reliability R to Target X_o with confidence C .

Given : That whatever number are tested, they are all run to a BOGEY X_1 without any failing.

Question : What sample size N_1 is needed to Bogey X_1 in order to have

$$R_C(X_o) = R \ ?$$

Needed : Weibull Slope b, and Minimum Life α.

PRINCIPLE EMPLOYED : EQUAL ENTROPY TOTALS FOR EQUAL PERFORMANCES

Thus, N_o run to X_o each must have the same ENTROPY TOTAL as N_1 run to X_1 each.

$$\therefore \quad N_o \left(\frac{X_o - \alpha}{\theta - \alpha} \right)^b = N_1 \left(\frac{X_1 - \alpha}{\theta - \alpha} \right)^b$$

$$\text{or} \quad N_o (X_o - \alpha)^b = N_1 (X_1 - \alpha)^b$$

$$\text{or} \quad N_1 = N_o \left(\frac{X_o - \alpha}{X_1 - \alpha} \right)^b$$

But , N_o is known to be $\dfrac{\ln (1-C)}{\ln R}$

$$\therefore \quad N_1 = \left(\frac{X_o - \alpha}{X_1 - \alpha} \right)^b \cdot \left(\frac{\ln (1 - C)}{\ln R} \right)$$

(SAMPLE SIZE FORMULA)

3. SPECIFIED PRECISION OF THE AVERAGE

Wanted : Mean correct within ± (100 P) % with ONE-SIDED CONFIDENCE C .

SAMPLE SIZE FORMULA :

$$N = \left(\frac{t_C \ \sigma}{P M} \right)^2$$

M = Sample Mean

σ = Population Standard Deviation (Assumed Known)

In case σ is unknown, take $\sigma_C = S \left(1 + \frac{t_C}{\sqrt{2N}} \right)$ in its place. (S = Sample Standard Deviation)

Then , the REQUIRED SAMPLE SIZE N is

$$N = \left(\frac{t_C S + \sqrt{t_C^2 S^2 + \frac{4 t_C^2 S P M}{\sqrt{2}}}}{2 P M} \right)^2$$

4 . SPECIFIED PRECISION OF A PERCENTILE

SAME AS (1) .

5 . SPECIFIED PRECISION OF THE STANDARD DEVIATION

Wanted : Standard Deviation accurate within $\pm$ (100 P) %
with ONE-SIDED CONFIDENCE C .

SAMPLE SIZE FORMULA : $$N = \frac{1}{2}\left(\frac{t_C}{P}\right)^2$$

t_C = C-level t-score in a normal distribution

6 . SPECIFIED PRECISION OF THE SKEWNESS

Wanted : SKEWNESS (α_3) accurate within $\pm$ (100 P) %
with ONE-SIDED CONFIDENCE C .

SAMPLE SIZE FORMULA : $$N = 6\left(\frac{t_C(1 + P)}{P\alpha_3}\right)^2$$

t_C = C-level t-score in normal distribution

α_3 = Sample Skewness

7 . SUFFICIENT CONFIDENCE IN A COMPARISON TEST

(a) MEAN LIFE COMPARISON

Confidence Desired : C (that B is better than A)

TOTAL DEGREES OF FREEDOM NEEDED :

$$T = \left(\frac{\ln \frac{1}{2(1 - C)}}{b \ln \rho}\right)^4$$

(ρ = Observed Mean Life Ratio)

(b = Weibull Slope at Mean)

EACH SAMPLE SIZE : $N = 1 + \left(\dfrac{\ln \dfrac{1}{2(1 - C)}}{b \ln \rho} \right)^2$

(b) B_{10} LIFE COMPARISON

EACH SAMPLE SIZE : $N = 10 \left(\dfrac{2 \hat{t}_c}{b \ln \rho} \right)^2$

ρ = Observed B_{10} Life Ratio

b = Weibull Slope at B_{10} Level

$\hat{t}_c$ = t-score to coincidence for confidence C

8 . SUFFICIENT ACCURACY IN THE WEIBULL SLOPE

WANTED : Weibull slope b correct within ± (100 P) % with a one-sided confidence of C .

ANALYSIS :

$$\sigma_b = \frac{b}{\sqrt{2N}} \quad \text{(Standard Error of the Weibull Slope)}$$

$$\therefore \; P b = t_C \, \sigma_b = \frac{t_C \, b}{\sqrt{2N}}$$

or

$$P = \frac{t_C}{\sqrt{2N}}$$

or

$$P^2 = \frac{t_C^2}{2N}$$

$$\therefore \; N = \frac{1}{2} \left(\frac{t_C}{P} \right)^2 \quad \text{(SAMPLE SIZE FORMULA)}$$

(NOTE: t_C is the C-level t-score in a NORMAL DISTRIBUTION)

(b = Sample Weibull Slope)